=23cm

Tutorial for solving the time-dependent Schrödinger equation with Scientific Workplace.

$$i\hbar\frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$$

Diagonalizing the Hamiltonian

$$\hat{H} = \begin{pmatrix} 0 & \frac{1}{2}\Omega \\ \frac{1}{2}\Omega & \Delta \end{pmatrix}$$
$$\hat{E} = \begin{pmatrix} \frac{1}{2}\Delta + \frac{1}{2}\sqrt{\Delta^2+\Omega^2} & 0 \\ 0 & \frac{1}{2}\Delta - \frac{1}{2}\sqrt{\Delta^2+\Omega^2} \end{pmatrix}$$
$$\hat{U} = \begin{pmatrix} -\frac{\Delta-\sqrt{\Delta^2+\Omega^2}}{\Omega} & -\frac{\Delta+\sqrt{\Delta^2+\Omega^2}}{\Omega} \\ 1 & 1 \end{pmatrix}$$

$$\hat{U}^{-1}\hat{H}\hat{U} = \begin{pmatrix} -\frac{\Delta-\sqrt{\Delta^2+\Omega^2}}{\Omega} & -\frac{\Delta+\sqrt{\Delta^2+\Omega^2}}{\Omega} \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 & \frac{1}{2}\Omega \\ \frac{1}{2}\Omega & \Delta \end{pmatrix} \begin{pmatrix} -\frac{\Delta-\sqrt{\Delta^2+\Omega^2}}{\Omega} & -\frac{\Delta+\sqrt{\Delta^2+\Omega^2}}{\Omega} \\ 1 & 1 \end{pmatrix} = \hat{E}$$

Solving the time-dependent Schrödinger equation

$$\begin{aligned} |\psi(t)\rangle &= e^{-i\hat{H}t/\hbar}|\psi(0)\rangle \\ &= e^{-i\hat{U}\hat{E}t\hat{U}^{-1}/\hbar}|\psi(0)\rangle \\ &= \hat{U}e^{-i\hat{E}t/\hbar}\hat{U}^{-1}|\psi(0)\rangle \end{aligned}$$

$$\begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = \begin{pmatrix} -\frac{\Delta-\sqrt{\Delta^2+\Omega^2}}{\Omega} & -\frac{\Delta+\sqrt{\Delta^2+\Omega^2}}{\Omega} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-it\left(\frac{1}{2}\Delta+\frac{1}{2}\sqrt{\Delta^2+\Omega^2}\right)} & 0 \\ 0 & e^{-it\left(\frac{1}{2}\Delta-\frac{1}{2}\sqrt{\Delta^2+\Omega^2}\right)} \end{pmatrix} \begin{pmatrix} -\frac{\Delta-\sqrt{\Delta^2+\Omega^2}}{\Omega} & -\frac{\Delta+\sqrt{\Delta^2+\Omega^2}}{\Omega} \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

etc..